## CMU 18-447 Introduction to Computer Architecture, Spring 2013 Midterm Exam 1

Date: Wed., 3/6

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## Instructions:

1. This is a closed book exam. You are allowed to have one letter-sized cheat sheet.
2. No electronic devices may be used.
3. This exam lasts 1 hour and 50 minutes.
4. Clearly indicate your final answer for each problem.
5. Please show your work when needed.
6. Please write your initials at the top of every page.
7. Please make sure that your answers to all questions (and all supporting work that is required) are contained in the space required.

Tips:

- Be cognizant of time. Do not spend to much time on one question.
- Be concise. You will be penalized for verbosity.
- Show work when needed. You will receive partial credit at the instructors' discretion.
- Write legibly. Show your final answer.

Initials:

## 1. Potpourri [65 points]

## (a) Full Pipeline [6 points]

Keeping a processor pipeline full with useful instructions is critical for achieving high performance. What are the three fundamental reasons why a processor pipeline cannot always be kept full?

Reason 1. Data dependences

Reason 2. Control flow dependences

Reason 3. Resource contention
(b) Exceptions vs. Interrupts [9 points]

In class, we distinguished exceptions from interrupts. Exceptions need to be handled when detected by the processor (and known to be non-speculative) whereas interrupts can be handled when convenient.

Why does an exception need to be handled when it is detected? In no more than 20 words, please.

The running program cannot continue if the exception is not immediately handled.

What does it mean to handle an interrupt "when it is convenient"?

The processor can handle the interrupt at any time.

Why can many interrupts be handled "when it is convenient"?

They are not needed for the running program's progress and not critical for system progress.
(c) Branch Target Buffer [5 points]

What is the purpose of a branch target buffer (in no more than 10 words, please)?

A BTB caches the target of a branch.

What is the downside of a design that does not use a branch target buffer? Please be concrete (and use less than 20 words).

Determining the branch target would cause bubbles in the pipeline.
(d) Return Address Prediction [4 points]

In lecture, we discussed that a return address stack is used to predict the target address of a return instruction instead of the branch target buffer. We also discussed that empirically a reasonablysized return address stack provides highly accurate predictions.

What key characteristic of programs does a return address stack exploit?

Usually, a return matches a function call.

Assume you have a machine with a 4-entry return address stack, yet the code that is executing has six levels of nested function calls each of which end with an appropriate return instruction. What is the return address prediction accuracy of this code?

```
4
```

(e) Restartable vs. Precise Interrupts [6 points]

As we discussed in one of the lectures, an exception (or interrupt) is "restartable" if a (pipelined) machine is able to resume execution exactly from the state when the interrupt happened and after the exception or interrupt is handled. By now you also should know what it means for an interrupt to be precise versus imprecise.

Can a pipelined machine have restartable but imprecise exceptions or interrupts?

Yes.

What is the disadvantage of such a machine over one that has restartable and precise exceptions or interrupts? Explain briefly.

It would be hard to debug code running on such a machine. Restartable exceptions do not ease debugging.
(f) Segmentation and Paging [4 points]

In segmentation, translation information is cached as part of the $\quad$ Segment selector
. In paging, translation information is cached in the $\square$
(g) Out-of-Order vs. Dataflow [8 points]

When does the fetch of an instruction happen in a dataflow processor?

When all inputs to the instruction are ready.

When does the fetch of an instruction happen in an out-of-order execution processor?

When the program counter points to that instruction.

In class, we covered several dataflow machines that implemented dataflow execution at the ISA level. These machines included a structure/unit called the "matching store." What is the function of the matching store (in less than 10 words)?

Determines whether a dataflow node is ready to fire.

What structure accomplishes a similar function in an out-of-order processor?

Reservation stations.

## (h) Tomasulo's Algorithm [5 points]

Here is the state of the reservation stations in a processor during a particular cycle ( $\times$ denotes an unknown value):

ADD Reservation Station

| Tag | V | Tag | Data | V | Tag | Data |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | D | $\times$ | 1 | $\times$ | 27 |
| B | 1 | $\times$ | 3 | 0 | E | $\times$ |
| C | 0 | B | $\times$ | 0 | A | $\times$ |
| $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ | $\times$ |

MUL Reservation Station

| Tag | V | Tag | Data | V | Tag | Data |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D | 0 | B | $\times$ | 0 | C | $\times$ |
| E | 1 | $\times$ | 16 | 0 | B | $\times$ |

What is wrong with this picture?

Cyclical dependences between instructions, which leads to deadlock. (Between tags B and E, and also between tags A, D, and C.)

## (i) Minimizing Stalls [10 points]

In multiple lectures, we discussed how the compiler can reorder instructions to minimize stalls in a pipelined processor. The goal of the compiler in these optimizations is to find independent instructions to place in between two dependent instructions such that by the time the consumer instruction enters the pipeline the producer would have produced its result.
We discussed that control dependences get in the way of the compiler's ability to reorder instructions. Why so?

At compile time, the compiler does not know whether control dependences are taken or not during execution. Hence, it does not know if an instruction can be moved above or below a branch.

What can the compiler do to alleviate this problem? Describe two solutions we discussed in class.

Solution 1. Predication. Eliminating branches solves the problem.

| Solution 2. | $\begin{array}{l}\text { Superblock or trace scheduling. The compiler profiles the code and deter- } \\ \text { mines likely branch directions and optimizes the instruction scheduling on } \\ \text { the frequently executed path. }\end{array}$ |
| :--- | :--- |

What is the major disadvantage or limitation of each solution?
Solution 1. Wasted instructions, ISA changes

Solution 2. Profile may not be accurate

## (j) Tomasulo's Algorithm Strikes Back [8 points]

You have a friend who is an architect at UltraFastProcessors, Inc. Your friend explains to you how their newest out-of-order execution processor that implements Tomasulo's algorithm and that uses full data forwarding works:
"After an instruction finishes execution in the functional unit, the result of the instruction is latched. In the next cycle, the tag and result value are broadcast to the reservation stations. Comparators in the reservation stations check if the source tags of waiting instructions match the broadcast tag and capture the broadcast result value if the broadcast tag is the same as a source tag."

Based on this description, is there an opportunity to improve the performance of your friend's design? Circle one:

## YES NO

If YES, explain what type of code leads to inefficient (i.e., lower performance than it could be) execution and why. (Leave blank if you answered NO above.)

Tag and result broadcast is delayed by a cycle, thus delaying the execution of dependent instructions.

If YES, explain what you would recommend to your friend to eliminate the inefficiency. (Leave blank if you answered NO above.)

Broadcast the tag and result as soon as an instruction finishes execution.

If NO, justify how the design is as efficient as Tomasulo's algorithm with full data forwarding can be. (Leave blank if you answered YES above.)

## BLANK

If NO, explain how the design can be simplified. (Leave blank if you answered YES above.)

## BLANK

## 2. Branch Prediction and Dual Path Execution [25 points]

Assume a machine with a 7 -stage pipeline. Assume that branches are resolved in the sixth stage. Assume that $20 \%$ of instructions are branches.
(a) How many instructions of wasted work are there per branch misprediction on this machine?

5 instructions.
(b) Assume $N$ instructions are on the correct path of a program and assume a branch predictor accuracy of $A$. Write the equation for the number of instructions that are fetched on this machine in terms of $N$ and $A$. (Please show your work for full credit.)

Note that if you assumed the wrong number of instructions in Part (a), you will only be marked wrong for this in Part (a). You can still get full credit on this and the following parts.

Correct path instructions $=N$
Incorrect path instructions $=N(0.2)(1-A) 5=N(1-A)$
Fetched instructions $=$ Correct path instructions + Incorrect path instructions

$$
\begin{aligned}
& =N+N(1-A) \\
& =N(2-A)
\end{aligned}
$$

(c) Let's say we modified the machine so that it used dual path execution like we discussed in class (where an equal number of instructions are fetched from each of the two branch paths). Assume branches are resolved before new branches are fetched. Write how many instructions would be fetched in this case, as a function of $N$. (Please show your work for full credit.)

Correct path instructions $=N$
Incorrect path instructions $=N(0.2) 5$
Fetched instructions $=$ Correct path instructions + Incorrect path instructions

$$
\begin{aligned}
& =N+N(1-0.8) 5 \\
& =2 N
\end{aligned}
$$

This solution assumes you have enough hardware in the frontend of the machine to fetch concurrently from both paths. If you assumed that both paths are fetched from on alternate cycles, that high-level approach is also OK, although note that you would need additional branch taken and not taken information to solve it completely.
(d) Now let's say that the machine combines branch prediction and dual path execution in the following way:

A branch confidence estimator, like we discussed in class, is used to gauge how confident the machine is of the prediction made for a branch. When confidence in a prediction is high, the branch predictor's prediction is used to fetch the next instruction; When confidence in a prediction is low, dual path execution is used instead.

Assume that the confidence estimator estimates a fraction $C$ of the branch predictions have high confidence, and that the probability that the confidence estimator is wrong in its high confidence estimation is $M$.

Write how many instructions would be fetched in this case, as a function of $N, A, C$, and $M$. (Please show your work for full credit.)

Correct path instructions $=N$
Incorrect path instructions due to...
lack of confidence $=N(0.2)(1-C) 5=N(1-C)$
incorrect high confidence estimate $=N(0.2) C M 5=N C M$
Fetched instructions $=$ Correct path instructions

+ Incorrect path instructions due to lack of confidence
+ Incorrect path instructions due to incorrect high confidence estimate
$=N+N(1-C)+N C M$
$=N[2+C(M-1)]$
Like above, if you assumed a different execution model for Part (c), you will not be penalized for using it in this part.

Initials:

## 3. Dataflow [20 points]

Here is a dataflow graph representing a dataflow program:


The following is a description of the nodes used in the dataflow graph:

| - | subtracts right input from left input |
| :---: | :--- |
| AND | bit-wise AND of two inputs |
| NOT | the boolean negation of the input (input and output are both boolean) |
| BR | passes the input to the appropriate output corresponding to the boolean condition |
| copy | passes the value from the input to the two outputs |
| $>0 ?$ | true if input greater than 0 |

Note that the input X is a non-negative integer.

What does the dataflow program do? Specify clearly in less than 15 words.

Calculates the parity of X . (True if the number of set bits in X is odd and false otherwise.)

## 4. Mystery Instruction [40 points]

That pesky engineer implemented yet another mystery instruction on the LC-3b. It is your job to determine what the instruction does. The mystery instruction is encoded as:

| 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1010 |  |  |  | DR |  |  | SR1 |  |  | 0 | 0 | 0 | 0 | 0 | 0 |

The modifications we make to the LC-3b datapath and the microsequencer are highlighted in the attached figures (see the next two pages). We also provide the original LC-3b state diagram, in case you need it. (As a reminder, the selection logic for SR2MUX is determined internally based on the instruction.)

The additional control signals are
GateTEMP1/1: NO, YES
GateTEMP2/1: NO, YES
LD.TEMP1/1: NO, LOAD
LD.TEMP2/1: NO, LOAD

ALUK/3: OR1 (A|0x1), LSHF1 (A $\ll 1$ ), PASSA, PASS0 (Pass value 0), PASS16 (Pass value 16)
COND/4:
COND 0000 ;Unconditional
COND 0001 ;Memory Ready
$\mathrm{COND}_{0010} ;$ Branch
COND $_{0011}$;Addressing mode
COND $_{0100}$;Mystery 1
$\mathrm{COND}_{1000} ;$ Mystery 2

The microcode for the instruction is given in the table below.

| State | Cond | J | Asserted Signals |
| :---: | :---: | :---: | :--- |
| $001010(10)$ | COND $_{0000}$ | 001011 | ALUK = PASS0, GateALU, LD.REG, <br> DRMUX = DR (IR[11:9] $)$ |
| $001011(11)$ | COND $_{0000}$ | 101000 | ALUK = PASSA, GateALU, LD.TEMP1, <br> SR1MUX = SR1 (IR[8:6]) |
| $101000(40)$ | COND $_{0000}$ | 110010 | ALUK = PASS16, GateALU, LD.TEMP2 |
| $110010(50)$ | COND $_{1000}$ | 101101 | ALUK = LSHF1, GateALU, LD.REG, <br> SR1MUX = DR, DRMUX = DR (IR[11:9]) |
| $111101(61)$ | COND $_{0000}$ | 101101 | ALUK = OR1, GateALU, LD.REG, <br> SR1MUX = DR, DRMUX = DR (IR[11:9]) |
| $101101(45)$ | COND $_{0000}$ | 111111 | GateTEMP1, LD.TEMP1 |
| $11111(63)$ | COND $_{0100}$ | 010010 | GateTEMP2, LD.TEMP2 |

Describe what this instruction does.

Bit-reverses the value in SR1 and puts it in DR.

Initials:

## Code:

```
State 10: DR \leftarrow 0
State 11: TEMP1 \leftarrow value(SR1)
State 40: TEMP2 \leftarrow 16
State 50: DR = DR << 1
    if (TEMP1[0] == 0)
        goto State 45
    else
        goto State 61
State 61: DR = DR | 0x1
State 45: TEMP1 = TEMP1 >> 1
State 63: DEC TEMP2
        if (TEMP2 == 0)
            goto State 18
        else
            goto State 50
```



Initials:


Address of Next State



## 5. Virtual Memory [40 points]

Suppose a $32 \mathrm{~K} \times 8 \mathrm{~K}$ matrix $A$ with 1-byte elements is stored in row major order in virtual memory. Assume only the program in question occupies space in physical memory. Show your work for full credit.

## Program 1

## Program 2

for (i = 0; i < 32768; i++)
for (j = 0; j < 8192; j++)

```
    for (j = 0; j < 8192; j++) for (i = 0; i < 32768; i++)
```

            A[i][j] = A[i][j] * A[i][j]; A[i][j] =A[i][j] * A[i][j];
    (a) If Program 1 yields 8 K page faults, what is the size of a page in this architecture?

A_SIZE $=32 \mathrm{~K} \times 8 \mathrm{~K} \times 1 \mathrm{~B}=256 \mathrm{MB}$
A_SIZE / PAGE_SIZE = PAGE_FAULTS
PAGE_SIZE $=$ A_SIZE $/$ PAGE_FAULTS $=256 \mathrm{MB} / 8 \mathrm{~K}=32 \mathrm{~KB}$

Assume the page size you calculated for the rest of this question.
(b) Consider Program 2. How many pages should the physical memory be able to store to ensure that Program 2 experiences the same number of page faults as Program 1 does?

8 K page faults is possible only if each page is brought into physical memory exactly once - i.e., there shouldn't be any swapping.

Therefore, the physical memory must be large enough to retain all the pages.
PAGE_COUNT $=$ A_SIZE $/$ PAGE_SIZE $=256 \mathrm{MB} / 32 \mathrm{~K}=8 \mathrm{~K}$
(c) Consider Program 2. How many page faults would Program 2 experience if the physical memory can store 1 page?
$32 \mathrm{~K} \times 8 \mathrm{~K} / 4=64 \mathrm{M}$. The inner loop touches a page four times before moving on to a different page.

What about if the physical memory can store 4 K pages?
$32 \mathrm{~K} \times 8 \mathrm{~K} / 4=64 \mathrm{M}$. After touching a page four times, the inner loop touches all other pages $(256 \mathrm{MB})$ before coming back to the same page.
(d) Now suppose the same matrix is stored in column-major order. And, the physical memory size is 32 MB .

How many page faults would Program 1 experience?
$32 \mathrm{~K} \times 8 \mathrm{~K}=256 \mathrm{M}$. After touching a page just once, the inner loop touches all other pages $(256 \mathrm{MB})$ before coming back to the same page.

How many page faults would Program 2 experience?

8 K . The inner loop touches all of a page and never comes back to the same page.
(e) Suppose still that the same matrix is stored in column-major order. However, this time the physical memory size is 8 MB .

How many page faults would Program 1 experience?
$32 \mathrm{~K} \times 8 \mathrm{~K}=256 \mathrm{M}$. After touching a page just once, the inner loop touches all other pages $(256 \mathrm{MB})$ before coming back to the same page.

How many page faults would Program 2 experience?

8K. The inner loop touches all of a page and never comes back to the same page.

## 6. Future File [40 points]

For this question, assume a machine with the following characteristics:

- Scalar, out-of-order dispatch with a 4-entry reorder buffer, future file, and full data forwarding.
- A 4-stage pipeline consisting of fetch, decode, execute, and writeback.
- Fetch and decode take 1 cycle each.
- Writeback takes 2 cycles and updates the future file and the reorder buffer.
- When the reorder buffer is filled up, fetch is halted.

A program that consists of three instructions: ADD, DIV, LD that have the following semantics:

- ADD $R d \leftarrow R s, ~ R t: A d d s$ the contents of Rs and Rt and stores the result in Rd.
- DIV Rd $\leftarrow$ Rs, Rt: Divides the contents of Rs by the contents of Rt and stores the result in Rd. Raises an exception if Rt is zero.
- LD Rd $\leftarrow$ Rs, Rt: Loads the contents of the base memory address Rs at the offset Rt and stores the result in Rd. Assume that calculated memory addresses are guaranteed to be 4-byte-aligned and the memory is bit-addressable.
An ADD instruction takes 1 cycle to execute, a DIV instruction takes 3 cycles to execute and a divide-by-zero exception, if present, is detected during the second cycle, and a LD instruction takes 5 cycles to execute.

Here is the state of the future file in the machine at the end of the cycle when a divide-by-zero exception is detected:
Future File

|  |  | V |
| :---: | :---: | :---: |
| Value |  |  |
| R1 | 0 | 21 |
| R2 | 1 | 13 |
| R3 | 1 | 0 |
| R4 | 1 | 3 |
| R5 | 1 | 25 |
| R6 | 1 | 1 |
| R7 | 1 | 17 |
| R8 | 1 | 8 |
| R9 | 1 | 9 |
| R10 | 0 | 23 |
| R11 | 1 | 7 |
| R12 | 1 | 19 |

Using what you know about the reorder buffer and the future file, fill in the missing contents of the reorder buffer in the machine. Assume reorder buffer entries are allocated from top to bottom in the diagram.

Reorder Buffer

| Oldest $\rightarrow$ | V | Exception? | Opcode | Rd | Rs | Rt | Dest. Value | Dest. Value Ready |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 0 | LD | R1 | R12 | R2* | ? | 0 |
|  | 1 | 0 | ADD | R4 | R3/? $\leftrightarrow$ | R4/R7 | 3 | 1 |
|  | 1 | 0 | ADD | R7 | R8 $\leftrightarrow$ | R9 | 17 | 1 |
| Youngest $\rightarrow$ | 1 | 1 | DIV | R10 | ? | R3 | ? | 0 |

Initials:
*Note that $\mathrm{R} 12+\mathrm{R} 2=32$, which is a valid 4-byte-aligned, bit addressable address.

## 7. Branch Prediction [35 points]

Assume the following piece of code that iterates through a large array populated with completely (i.e., truly) random positive integers. The code has four branches (labeled B1, B2, B3, and B4). When we say that a branch is taken, we mean that the code inside the curly brackets is executed.

```
for (int i=0; i<N; i++) { /* BI */
    val = array[i]; /* TAKEN PATH for B1 */
    if (val % 2 == 0) { /* B2 */
        sum += val; /* TAKEN PATH for B2 */
    }
    if (val % 3 == 0) { /* B3 */
        sum += val; /* TAKEN PATH for B3 */
    }
    if (val % 6 == 0) { /* B4 */
        sum += val; /* TAKEN PATH for B4 */
    }
}
```

(a) Of the four branches, list all those that exhibit local correlation, if any.

Only B1.

B2, B3, B4 are not locally correlated. Just like consecutive outcomes of a die, an element being a multiple of $N(N$ is 2,3 , and 6 , respectively for B2, B3, and B4) has no bearing on whether the next element is also a multiple of $N$.
(b) Which of the four branches are globally correlated, if any? Explain in less than 20 words.

B4 is correlated with B2 and B3. 6 is a common multiple of 2 and 3 .

Now assume that the above piece of code is running on a processor that has a global branch predictor. The global branch predictor has the following characteristics.

- Global history register (GHR): 2 bits.
- Pattern history table (PHT): 4 entries.
- Pattern history table entry (PHTE): 11-bit signed saturating counter (possible values: -10241023)
- Before the code is run, all PHTEs are initially set to 0 .
- As the code is being run, a PHTE is incremented (by one) whenever a branch that corresponds to that PHTE is taken, whereas a PHTE is decremented (by one) whenever a branch that corresponds to that PHTE is not taken.
(d) After 120 iterations of the loop, calculate the expected value for only the first PHTE and fill it in the shaded box below. (Please write it as a base-10 value, rounded to the nearest one's digit.)

Hint. For a given iteration of the loop, first consider, what is the probability that both B1 and B2 are taken? Given that they are, what is the probability that B3 will increment or decrement the PHTE? Then consider...

Show your work.

Without loss of generality, let's take a look at the numbers from 1 through 6 . Given that a number is a multiple of two (i.e., $2,4,6$ ), the probability that the number is also a multiple of three (i.e., 6) is equal to $1 / 3$, let's call this value Q. Given that a number is a multiple of two and three (i.e., 6), the probability that the number is also a multiple of six (i.e., 6) is equal to 1 , let's call this value R .

For a single iteration of the loop, the PHTE has four chances of being incremented/decremented, once at each branch.

- B3's contribution to PHTE. The probability that both B1 and B2 are taken is denoted as $\mathrm{P}\left(\mathrm{B} 1 \_\mathrm{T} \& \& \mathrm{~B} 2 \_\mathrm{T}\right)$, which is equal to $\mathrm{P}\left(\mathrm{B} 1 \_\mathrm{T}\right) * \mathrm{P}\left(\mathrm{B} 2 \_\mathrm{T}\right)=1^{*} 1 / 2=1 / 2$. Given that they are, the probability that B 3 is taken, is equal to $\mathrm{Q}=1 / 3$. Therefore, the PHTE will be incremented with probability $1 / 2^{*} 1 / 3=1 / 6$ and decremented with probability $1 / 2^{*}(1-1 / 3)=1 / 3$. The net contribution of B3 to PHTE is $1 / 6-1 / 3=-1 / 6$.
- B4's contribution to PHTE. $\mathrm{P}\left(\mathrm{B} 2 \_\mathrm{T} \& \& \mathrm{~B} 3 \_\mathrm{T}\right)=1 / 6 . \mathrm{P}\left(\mathrm{B} 4 \_\mathrm{T} \mid \mathrm{B} 2 \_\mathrm{T} \& \& \mathrm{~B} 3 \_\mathrm{T}\right)=$ $\mathrm{R}=1$. B4's net contribution is $1 / 6^{*} 1=1 / 6$.
- B1's contribution to PHTE. P(B3_T \&\& B4_T $)=1 / 6 . \mathrm{P}\left(\mathrm{B} 1 \_\mathrm{T} \mid \mathrm{B} 3 \_\mathrm{T} \& \& \mathrm{~B} 4 \_\mathrm{T}\right)=$ 1. B1's net contribution is $1 / 6^{*} 1=1 / 6$.
- B2's contribution to PHTE. P(B4_T \&\& B1_T) $=1 / 6^{*} 1=1 / 6$. P(B2_T | B4_T \&\& $\left.\mathrm{B} 1 \_\mathrm{T}\right)=1 / 2$. B2's net contribution is $1 / 6^{*} 1 / 2-1 / 6^{*} 1 / 2=0$.

For a single iteration, the net contribution to the PHTE, summed across all the four branches, is equal to $1 / 6$. Since there are 120 iterations, the expected PHTE value is equal to $1 / 6^{*} 120=\mathbf{2 0}$.


## 8. Bonus (Question 7 Continued) [45 points]

(a) Assume the same question in Part (d) of Question 7. Your job in this question is to fill in the rest of the PHTEs. In other words, after 120 iterations of the loop in Question 7, calculate the expected value for the rest of the PHTEs (i.e., PHTEs 2, 3, 4) and fill in the PHT below. (Please write them as base-10 values, rounded to the nearest one's digit.)

Show your work.

## PHTE2: TN

$\mathrm{P}\left(\mathrm{B} 1 \_\mathrm{T} \& \& \mathrm{~B} 2 \_\mathrm{N}\right)=1 / 2 . \mathrm{P}\left(\mathrm{B} 3 \_\mathrm{T} \mid \mathrm{B} 1 \_\mathrm{T} \& \& \mathrm{~B} 2 \_\mathrm{N}\right)=1 / 3 . \quad \Delta \mathrm{PHTE}=1 / 2^{*}(1 / 3-2 / 3)=-1 / 6$.
$\mathrm{P}\left(\mathrm{B} 2 \_\mathrm{T} \& \& \mathrm{~B} 3-\mathrm{N}\right)=1 / 3 . \mathrm{P}\left(\mathrm{B} 4-\mathrm{T} \mid \mathrm{B} 2 \_\mathrm{T} \& \& \mathrm{~B} 3 \_\mathrm{N}\right)=0 . \Delta \mathrm{PHTE}=1 / 3^{*}-1=-1 / 3$.
$\mathrm{P}\left(\mathrm{B} 3 \_\mathrm{T} \& \& \mathrm{~B} 4 \_\mathrm{N}\right)=1 / 6 . \mathrm{P}\left(\mathrm{B} 1 \_\mathrm{T} \mid \mathrm{B} 3 \_\mathrm{T} \& \& \mathrm{~B} 4 \_\mathrm{N}\right)=1 . \Delta \mathrm{PHTE}=1 / 6^{*} 1=1 / 6$.
$\mathrm{P}\left(\mathrm{B} 4 \_\mathrm{T} \& \& \mathrm{~B} 1 \_\mathrm{N}\right)=0 . \mathrm{P}\left(\mathrm{B} 2 \_\mathrm{T} \mid \mathrm{B} 4 \_\mathrm{T} \& \& \mathrm{~B} 1 \_\mathrm{N}\right)=\mathrm{X} . \Delta \mathrm{PHTE}=0$.
Answer: $120 *(-1 / 6-1 / 3+1 / 6+0)=-40$

## PHTE3: NT

$\mathrm{P}\left(\mathrm{B} 1 \_\mathrm{N}\right.$ \&\& B2_T $)=0 . \mathrm{P}\left(\mathrm{B} 3 \_\mathrm{T} \mid \mathrm{B} 1 \_\mathrm{N} \& \& \mathrm{~B} 2 \_\mathrm{T}\right)=\mathrm{X} . \Delta \mathrm{PHTE}=0$.
$\mathrm{P}\left(\mathrm{B} 2 \_\mathrm{N}\right.$ \&\& B3_T $)=1 / 6 . \mathrm{P}\left(\mathrm{B} 4 \_\mathrm{T} \mid \mathrm{B} 2 \_\mathrm{N} \& \& \mathrm{~B} 3 \_\mathrm{T}\right)=0 . \Delta \mathrm{PHTE}=1 / 6^{*}-1=-1 / 6$.
$\mathrm{P}\left(\mathrm{B} 3 \_\mathrm{N}\right.$ \&\& B4_T $)=0$. $\mathrm{P}\left(\mathrm{B} 1 \_\mathrm{T} \mid \mathrm{B} 3 \_\mathrm{N} \& \& \mathrm{~B} 4 \_\mathrm{T}\right)=\mathrm{X} . \Delta \mathrm{PHTE}=0$.
$\mathrm{P}\left(\mathrm{B} 4 \_\mathrm{N} \& \& \mathrm{~B} 1 \_\mathrm{T}\right)=5 / 6 . \mathrm{P}\left(\mathrm{B} 2 \_\mathrm{T} \mid \mathrm{B} 4 \_\mathrm{N} \& \& \mathrm{~B} 1 \_\mathrm{T}\right)=1 / 2 . \Delta \mathrm{PHTE}=5 / 6^{*}(1 / 2-1 / 2)=0$.
Answer: $120^{*}(0-1 / 6+0+0)=\mathbf{- 2 0}$

## PHTE4: NN

$\mathrm{P}\left(\mathrm{B} 1 \_\mathrm{N}\right.$ \&\& B2_N $)=0 . \mathrm{P}\left(\mathrm{B} 3 \_\mathrm{T} \mid \mathrm{B} 1 \_\mathrm{N}\right.$ \&\& B2_N $)=\mathrm{X}$. PHTE_DELTA=0.
$\mathrm{P}\left(\mathrm{B} 2 \_\mathrm{N}\right.$ \&\& B3_N $)=1 / 3$. $\mathrm{P}\left(\mathrm{B} 4 \_\mathrm{T} \mid \mathrm{B} 2 \_\mathrm{N}\right.$ \&\& B3_N $)=0$. PHTE_DELTA $=1 / 3^{*}-1=-1 / 3$.
$\mathrm{P}\left(\mathrm{B} 3 \_\mathrm{N}\right.$ \&\& B4_N $)=2 / 3$. $\mathrm{P}\left(\mathrm{B} 1 \_\mathrm{T} \mid \mathrm{B} 3 \_\mathrm{N} \& \& \mathrm{~B} 4 \_\mathrm{N}\right)=1$. PHTE_DELTA $=2 / 3^{*} 1=2 / 3$.
$\mathrm{P}\left(\mathrm{B} 4 \_\mathrm{N}\right.$ \&\& $\left.\mathrm{B} 1 \_\mathrm{N}\right)=0 . \mathrm{P}\left(\mathrm{B} 2 \_\mathrm{T} \mid \mathrm{B} 4 \_\mathrm{N} \& \& \mathrm{~B} 1 \_\mathrm{N}\right)=\mathrm{X}$. PHTE_DELTA $=0$.
Answer: $120 *(0-1 / 3+2 / 3+0)=40$.

(b) After the first 120 iterations, let us assume that the loop continues to execute for another 1 billion iterations. What is the accuracy of this global branch predictor during the 1 billion iterations? (Please write it as a percentage, rounded to the nearest single-digit.)

Show your work.

```
Given a history of TT, the number of correct predictions per
iteration =
P}(\textrm{B}1_T && B2_T) * P(B3_T | B1_T && B2_T) +
P}(\textrm{B}2_T && B3_T) * P(B4_T | B2_T && B3_T) +
P}(\textrm{B}3_T && B4_T) * P(B1_T | B3_T && B4_T) +
P}(\textrm{B}4\_\textrm{T}&& B1_T) * P(B2_T | B4_T && B1_T) = 7/12
Given a history of TN, the number of correct predictions per
iteration =
P(B1_T && B2_N) * P(B3_N | B1_T && B2_N) +
P}(\textrm{B}2_N && B3_N) * P(B4_N | B2_T && B3_N) +
P}(\textrm{B}3_N && B4_N) * P(B1_N | B3_T && B4_N) +
P(B4_N && B1_N) * P(B2_N | B4_T && B1_N) = 2/3
Given a history of NT, the number of correct predictions per
iteration = 7/12
Given a history of NN, the number of correct predictions per
iteration = 2/3
```

Correct predictions per iteration $=7 / 12+2 / 3+7 / 12+2 / 3=30 / 12$
Branches per iteration $=4$
Accuracy $=(30 / 12) / 4=30 / 48=5 / 8=62.5 \%=\mathbf{6 3 \%}$
(c) Without prior knowledge of the contents of the array, what is the highest accuracy that any type of branch predictor can achieve during the same 1 billion iterations as above? (Please write it as a percentage, rounded to the nearest single-digit.)

Show your work.

Per-branch accuracy:
B1: $100 \%$
B2: $50 \%$ (half the numbers are even)
B3: $33 \%$ (a third of the numbers are a multiple of three)
B4: 100\% (global correlation)

Average accuracy: $70.8 \%=\mathbf{7 1 \%}$

Initials:

Stratchpad

Initials:

Stratchpad

Initials:

Stratchpad

Initials:

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Initials:

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